Contour integrals.

Reminder:
- A function \( f \) defined in a punctured neighborhood of \( z_0 \) is said to have a pole at \( z_0 \) if \( \frac{1}{f(z)} \) is holomorphic in a full neighborhood of \( z_0 \) and has a zero at \( z_0 \).
- If \( f \) has a pole at \( z_0 \) then in a neighborhood of \( z_0 \) there exists a nonvanishing holomorphic function \( h \) and a unique \( n > 0 \) such that \( f(z) = \frac{h(z)}{(z-z_0)^n} \). The number \( n \) is called the order of the pole \( z_0 \).
- If \( f \) has a pole of order \( n \) at \( z_0 \) then \( f(z) = a_{-n}(z-z_0)^n + \ldots + a_{-1}(z-z_0) + g(z) \), where \( g \) is holomorphic in a neighborhood of \( z_0 \). Coefficient \( a_{-1} \) is called the residue of \( f \) at \( z_0 \):
  \[ \text{Res}_{z=z_0} f(z) = a_{-1}. \]
- Let \( f \) be a continuous function in the closure of \( \Omega \subset \mathbb{C} \) and holomorphic in \( \Omega \) except for a finite number of poles \( z_k \in \Omega \). Then
  \[ \oint_{\partial \Omega} f(z) \, dz = 2\pi i \sum_{k=1}^{n} \text{Res}_{z=z_k} f(z), \]
  where \( \partial \Omega \) is oriented such that domain \( \Omega \) is on its left.

1. Calculate residues at poles of \( f(z) \):
   - (a) \( f(z) = \frac{z^n}{(z^2+1)(z+1)^n} \);
   - (b) \( f(z) = z^{11}e^{1/z^2} \);
   - (c) \( f(z) = \frac{1}{z\sin z^2} \).

2. Calculate the following contour integrals:
   - (a) \( \oint_{|z|=3} \frac{dz}{(z-1)^n(1-\cos(z))} \);
   - (b) \( \oint_{|z|=7} \frac{1-\text{ch}(z)}{z^3+4z+2} \, dz \).

3. Calculate the integrals:
   - (a) \( \int_{-\infty}^{+\infty} \frac{x^2 \, dx}{(x^2+1)(x^2+2)} \);
   - (b) \( \int_{0}^{+\infty} \frac{x^n \, dx}{(x^2+a^2)^2}, \ a > 0; \)
   - (c) \( \int_{-\infty}^{+\infty} \frac{\cos x \, dx}{(x^2+4x+2)^2}. \)